## REFERENCES

1. G. P. Cressman, "On the Forecasting of Long Waves in the Upper Westerlies," *Journal of Meteorology*, vol. 5, No. 2, Apr. 1948, pp. 44–57.
2. G. E. Forsythe, "A Generalization of the Thermal Wind Equation for Arbitrary Horizontal Flow," *Bulletin of the American Meteorological Society*, vol. 26, No. 10, Dec. 1945, pp. 371–375.
3. T. N. Krishnamurti, "The Subtropical Jet Stream of Winter," *Journal of Meteorology*, vol. 18, No. 2, Apr. 1961, pp. 172–191.
4. K. Mohri, "On the Fields of Wind and Temperature over Japan and Adjacent Waters During Winter of 1950–51," *Tellus*, vol. 5, No. 3, Aug. 1953, pp. 340–355.
5. K. Mohri, "Jet Stream and Upper Fronts in the General Circulation and their Characteristics over the Far East," *Geophysical Magazine* (Tokyo), vol. 29, No. 1, Nov. 1958, pp. 45–126 (part I), and vol. 29, No. 3, Aug. 1959, pp. 333–412 (part II).
6. C. W. Newton, "Variations of Frontal Structure of Upper Level Troughs," *Geophysica*, vol. 6 (Palmén 60th anniversary volume), Nos. 3–4, Aug. 1958, pp. 357–375.
7. C. W. Newton and A. V. Persson, "Structural Characteristics of the Subtropical Jet Stream and Certain Lower Stratospheric Wind Systems," *Tellus*, vol. 14, No. 2, May 1962, pp. 221–241.
8. C. W. Newton and E. Palmén, "Kinematic and Thermal Properties of a Large-Amplitude Wave in the Westerlies," *Tellus*, vol. 15, No. 2, May 1963, pp. 99–119.
9. G. W. Platzman, "Some Remarks on the Measurement of Curvature and Vorticity," *Journal of Meteorology*, vol. 4, No. 2, Apr. 1947, pp. 58–62.
10. H. Riehl and Collaborators, "Forecasting in Middle Latitudes," *Meteorological Monographs*, American Meteorological Society, Boston, vol. 1, No. 5, June 1952, 80 pp.
11. W. J. Saucier and Collaborators, "Wind Field Near the Tropopause," *Final Report*, Contract AF 19(604)–1565, Agricultural and Mechanical College of Texas, December 1958, 140 pp.


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# CORRESPONDENCE

## Comments on "Some Problems Involved in the Numerical Solutions of Tidal Hydraulics Equations"

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In an article by Harris and Jelesnianski [1] several references were made to a finite difference approximation to the hydrodynamic equations which I have employed for tidal and wind-stress computations in the North Sea [2]. In the same notation as in [1], the scheme reads as follows:

$$U^{m+1}_{i,j}=U^m_{i,i}-\frac{g\Delta t}{2\Delta s}D_{i,j}[h^m_{i+1,j}-h^m_{i-1,j}]+\Delta t\,[fV^m_{i,j}]$$

$$V^{m+1}_{i,j}=V^m_{i,j}-\frac{g\Delta t}{2\Delta s}D_{i,j}[h_{i,j+1}-h_{i,j-1}]-\Delta t\,[fU^m_{i,j}]$$

$$h^{m+1}_{i,j}=h^m_{i,j}-\frac{\Delta t}{2\Delta s}[U^{m+1}_{i+1,j}-U^{m+1}_{i-1,j}+V^{m+1}_{i,j+1}-V^{m+1}_{i,j-1}] \quad (1)$$

This scheme is stable if $f=0$ provided $\Delta t<\sqrt{2/(gD)}\Delta s$ but unfortunately gives rise to growing solutions if rotation is involved. (The maximum eigenvalue is $|\lambda_{\max}|=\sqrt{1+f^2\Delta t^2}$). Instability was indeed observed by the authors in a test computation (fig. 2 in [1]).

Since scheme (1) has the great advantage, in contrast to the commonly used central difference scheme ((16), (17), (18) in [1]), that only field values at time $(m-1)$ $\Delta t$ are required it seems worthwhile to try to remove the shortcomings of scheme (1). This is easily done if in (1) the Coriolis terms are centered in time so that $f\Delta tU^m_{i,j}$ and $f\Delta tV^m_{i,j}$ are replaced by $\frac{1}{2}(f\Delta tU^{m+1}_{i,j}+f\Delta tU^m_{i,j})$ and $\frac{1}{2}(f\Delta tV^{m+1}_{i,j}+f\Delta tV^m_{i,j})$ respectively. In this way the scheme becomes implicit. If a grid is used, however, where both $U$ and $V$ are defined at the same grid points, only a system of two algebraic equations has to be solved for the unknowns $U^{m+1}_{i,j}$ and $V^{m+1}_{i,j}$. Thus the new scheme can be written

$$U^{m+1}_{i,j}-\frac{f\Delta t}{2}V^{m+1}_{i,j}=U^m_{i,j}-\frac{g\Delta t}{2\Delta s}D_{i,j}[h_{i+1,j}-h_{i-1,j}]+\frac{f\Delta t}{2}V^m_{i,j}\equiv X^m_{i,j}$$

$$V^{m+1}_{i,j}+\frac{f\Delta t}{2}U^{m+1}_{i,j}=V^m_{i,j}-\frac{g\Delta t}{2\Delta s}D_{i,j}[h_{i,j+1}-h_{i,j-1}]-\frac{f\Delta t}{2}U^m_{i,j}\equiv Y^m_{i,j}$$

$$U^{m+1}_{i,j}=\frac{1}{1+\frac{f^2\Delta t^2}{4}}\left[X^m_{i,j}+\frac{f\Delta t}{2}Y^m_{i,j}\right] \quad (2)$$

$$V^{m+1}_{i,j}=\frac{1}{1+\frac{f^2\Delta t^2}{4}}\left[Y^m_{i,j}-\frac{f\Delta t}{2}X^m_{i,j}\right]$$

$$h^{m+1}_{i,j}=h^m_{i,j}-\frac{\Delta t}{2\Delta s}[U^{m+1}_{i+1,j}-U^{m+1}_{i-1,j}+V^{m+1}_{i,j+1}-V^{m+1}_{i,j-1}]$$

Now the stability condition reads $\Delta t < \sqrt{2/(gD)}\Delta s$ whatever value $f$ has; then all eigenvalues are exactly one in absolute value. Thus scheme (2) possesses neutral stability.

A more symmetric form of (2) is obtained if the time level of $h$ is shifted forward by $\Delta t/2$. Then for all times $h^m_{i,j}$ and $h^{m+1}_{i,j}$ in (2) have to be replaced by $h^{m+1/2}_{i,j}$ and $h^{m+3/2}_{i,j}$ respectively. As a consequence of this procedure each equation is centered in time (but is not identical to the central difference scheme) whereas the stability condition remains unchanged.

The scheme (2) has second order accuracy in $\Delta t$ and $\Delta x$.

### REFERENCES


1. D. L. Harris and C. P. Jelesnianski, "Some Problems Involved in the Numerical Solutions of Tidal Hydraulics Equations," *Monthly Weather Review*, vol. 92, No. 9, Sept. 1964, pp. 409–422.
2. G. Fischer, "Ein numerisches Verfahren zur Errechnung von Windstau und Gezeiten in Randmeeren," *Tellus*, vol. 11, No. 1, Feb. 1959, pp. 60–76.


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## Reply

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The comment by Günter Fischer on our paper is appreciated. We did not consider the form proposed by his equations (2) and have not tested them. Judging from the stability analysis presented by Fischer, we believe that his equations (2) may be satisfactory for most problems actually considered in [1], but not for the problems we were leading to.

The computational stability conditions which he and we have discussed, have been established only for homogeneous equations, with constant coefficients and periodic or cyclical boundary conditions. Our problem, the computation of storm surges on the open coast does not satisfy any of these restrictions. For example, it is necessary to consider basins with depths that vary considerably. Thus the computational stability theorems can be used only as a guide, whereas the actual stability must be established by sample calculations. One such test is to require that the energy of the computed flow does not grow faster than energy supplied to the system. This test could not be readily applied to equations (1) since the mixture of forward and backward differences led to a phase lag between the velocity and height fields; that is, to a phase shift between the kinetic and potential energy. The result was an oscillation in the total energy.

This disadvantage could be eliminated when the period of the disturbance is well known, but we doubt that this is true for the types of disturbances we wish to investigate.

### REFERENCE


1. D. L. Harris and C. P. Jelesnianski, "Some Problems Involved in the Numerical Solutions of Tidal Hydraulics Equations," *Monthly Weather Review*, vol. 92, No. 9, Sept. 1964, pp. 409–422.